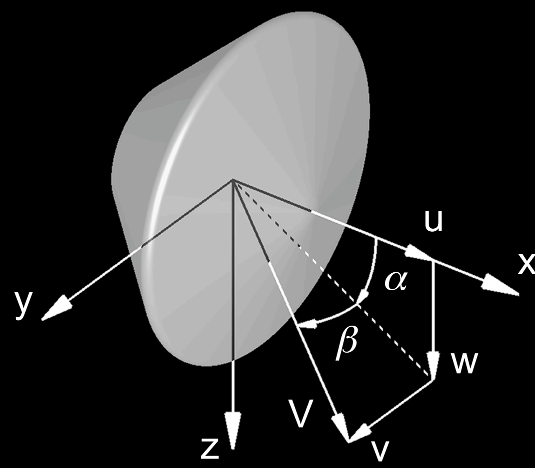

Flight Mechanics

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Version 6

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Flight Mechanics

Definition
The branch of engineering that studies the motion of aerospace vehicles in flight when acted upon by gravitational, aerodynamic, propulsive, and other external forces.

In this set of lectures we will focus on the flight mechanics of entry and descent vehicles, with an emphasis on:

- deriving the necessary differential equations
- modeling gravity and aerodynamic forces
- numerically integrating the differential equations

Basic Assumptions

- Single, rigid, flight vehicle <u>without</u> rotating masses
- Constant mass properties (i.e., mass, moments of inertia, products of inertia, center of mass)
- The planet is flat and non-rotating (planet surface is used to define an inertial coordinate system)
- No wind
- The the atmospheric density, ρ, is a function of the altitude, h

Additional assumptions are listed throughout the lecture.

Coordinate Systems

Three coordinate systems are used in this lecture

Body Coordinate System

Planet Coordinate System

Vehicle-Carried Planet Coordinate System

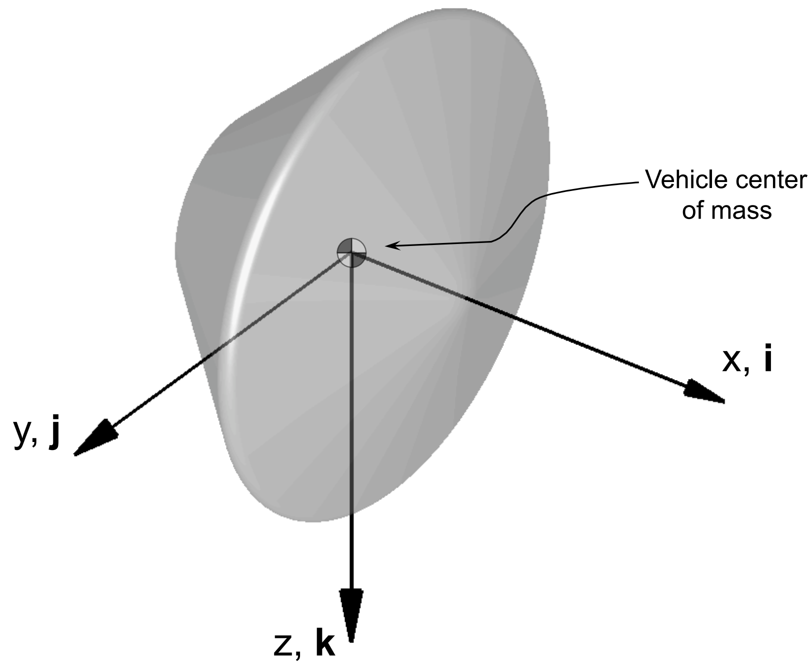
Coordinate Systems

Body Coordinate System

- Cartesian (right-hand)
- Fixed to the vehicle
- Origin at the center of mass. This assumption is very important. The equations of motion have additional terms not presented in these lecture notes if the origin is not at the center of mass.
- Axes: (x, y, z)
- Unit vectors: (**i**, **j**, **k**)
- Airplane convention is:
  - x axis forward
  - z axis "down"
  - y axis out right wing
  - if possible, y = constant, is plane of symmetry (i.e., x-z plane)
- Not inertial

Coordinate Systems

Body Coordinate System



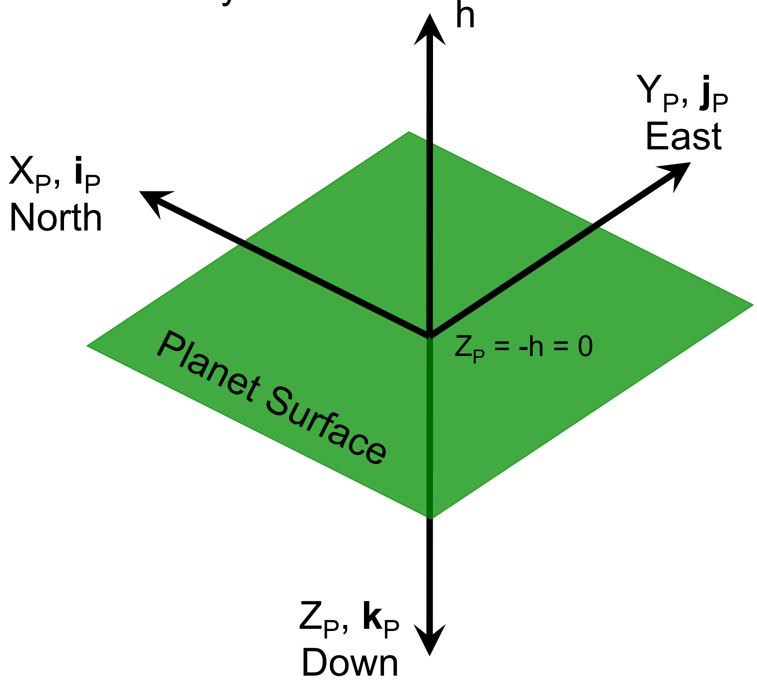
Coordinate Systems

Planet Coordinate System

- Cartesian (right-hand)
- Origin fixed at a specific location on the surface of the planet (i.e., at h = 0)
- Axes: (X_P, Y_P, Z_P)
- Unit vectors: $(\mathbf{i}_P, \mathbf{j}_P, \mathbf{k}_P)$
- Z_P axis down (i.e., $h = -Z_P$)
- $Z_P = 0$ is surface of the planet (i.e., X_P-Y_P plane)
- X_P axis points north
- Y_P axis points east
- Assumed to be inertial

Coordinate Systems

Planet Coordinate System



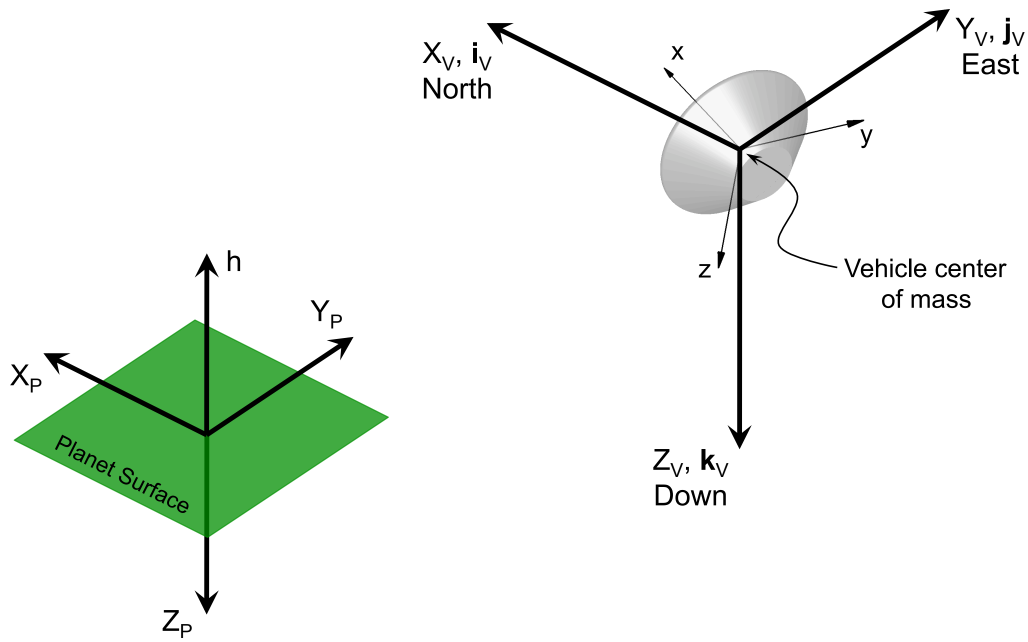
Coordinate Systems

Vehicle-Carried Planet Coordinate System

- Cartesian (right-hand)
- Origin at the vehicle center of mass (i.e., coincident with the origin of the body coordinate system)
- Axes: (X_V, Y_V, Z_V)
- Unit vectors: $(\mathbf{i}_V, \mathbf{j}_V, \mathbf{k}_V)$
- Axes parallel to the planet coordinate system
  $X_V \parallel X_P$ ➔ X_V axis points north
  $Y_V \parallel Y_P$ ➔ Y_V axis points east
  $Z_V \parallel Z_P$ ➔ Z_V axis points down
- Not inertial

Coordinate Systems

Vehicle-Carried Planet Coordinate System



State Variables

The <u>twelve</u> state variables that completely define the motion of the vehicle are:

u, v, w - components of the vehicle center of mass velocity vector, **V**, in the body coordinate system

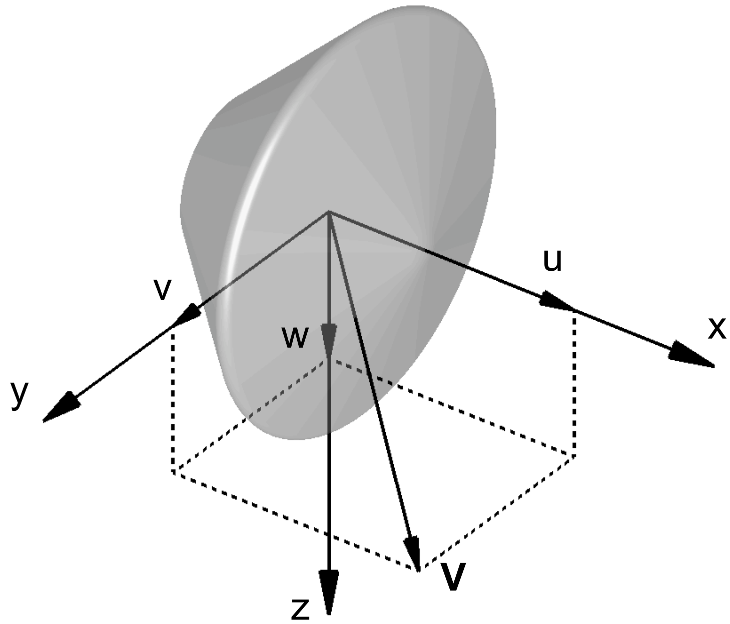
$$\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k} \tag{1}$$

p, q, r - components of the vehicle rotation rate vector, **Ω**, in the body coordinate system

$$\boldsymbol{\Omega} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k} \tag{2}$$

State Variables

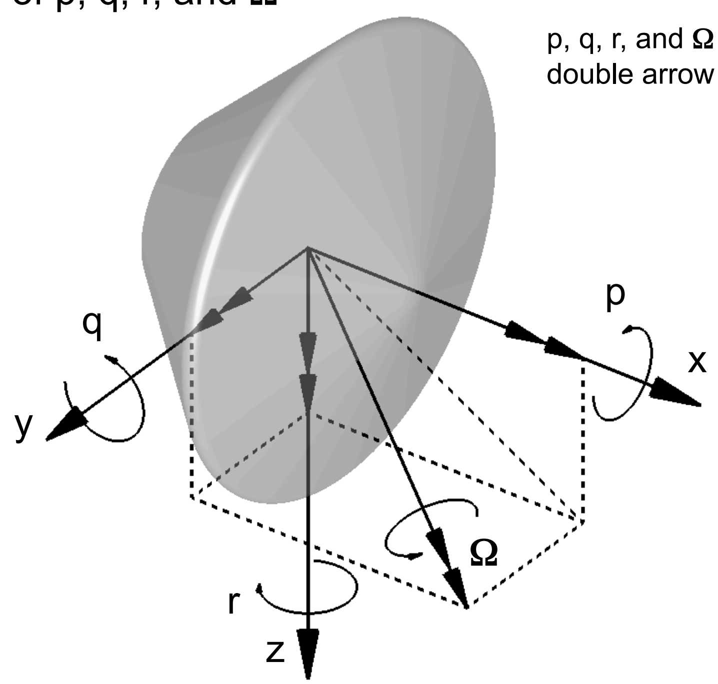
Definition of u, v, w, and **V**



State Variables

Definition of p, q, r, and **Ω**

p, q, r, and **Ω** are shown by double arrow <u>and</u> arc arrow



State Variables

ψ, θ, ϕ - *Euler Angles* defining the relative orientation between the vehicle body coordinate system (x, y, z), the planet coordinate system (X_P, Y_P, Z_P), and the vehicle-carried planet coordinate system (X_V, Y_V, Z_V)

The rotation sequence is specified as

ψ (azimuth or yaw) about the z-axis

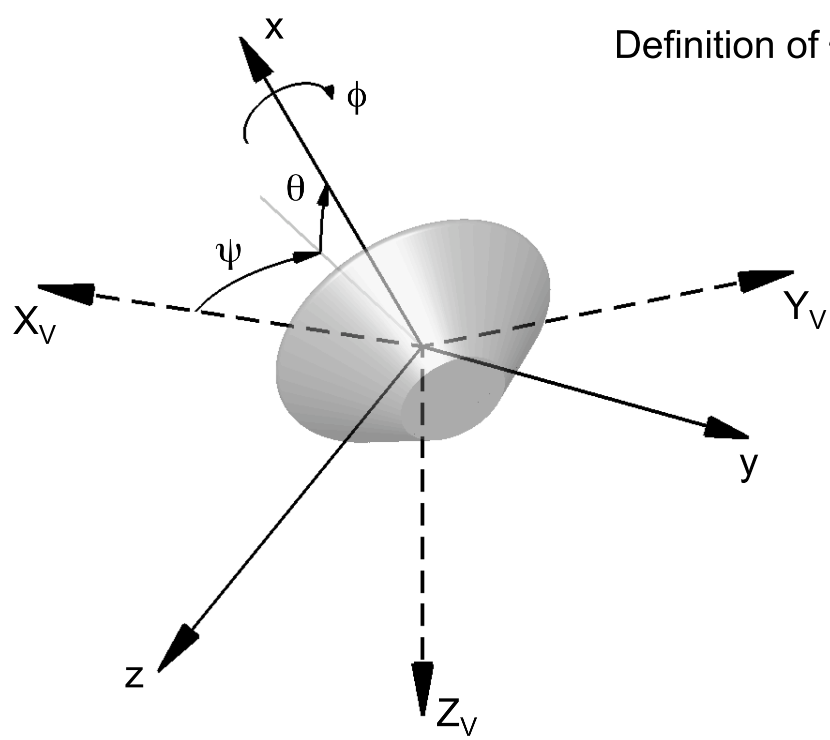
θ (elevation or pitch) about the y-axis

ϕ (bank or roll) about the x-axis

Euler Angles are easily visualized. However, they have problems with singularities. These problems can be addressed by using *Euler Parameters* (aka *quaternions*) to express the relative orientations between the vehicle body coordinate system (x, y, z) and the vehicle-carried planet coordinate system (X_V, Y_V, Z_V).

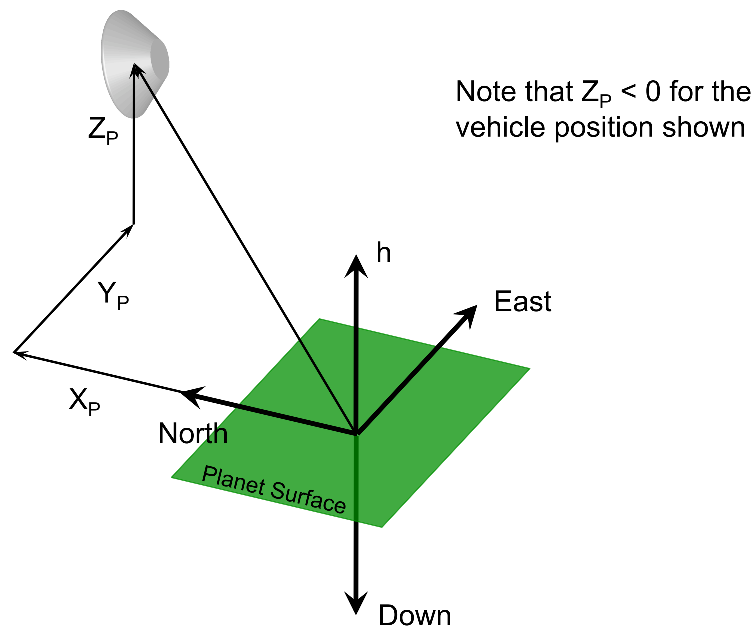
State Variables

Definition of ψ, θ, and ϕ



State Variables

X_P, Y_P, Z_P - position of the vehicle's center of mass in the planet coordinate system



Equations of Motion

One first-order differential equation is needed for each state variables to fully characterize the vehicle's motion. Thus, 12 first-order differential equations are needed. The equations can be divided into two groups:

Kinetic Equations of Motion - derived from Newton's 2nd law and Euler's Laws [6 equations: 3 force (u, v, w) - Newton, and 3 moment (p, q, r) - Euler]

Kinematic Equations of Motion - describe relationships between motion components that do not depend on forces and moments [6 equations: (ψ, θ, ϕ) and (X_P, Y_P, Z_P)]

Equations of Motion - u, v, w

The linear momentum vector of the vehicle, **p**, is defined by

$$\mathbf{p} = m\mathbf{V} \tag{3}$$

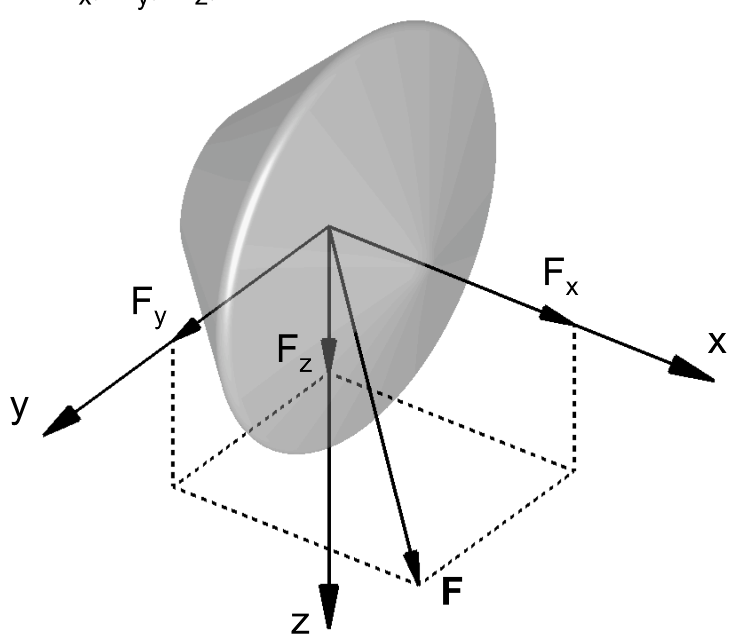
where m is the vehicle's mass. From Newton's second law

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} + \boldsymbol{\Omega} \times \mathbf{p} \tag{4}$$

Notice the additional term **Ω** x **p**. This term arises because the body coordinate system is rotating. **F** is the external forces vector (e.g., gravitational, aerodynamic, buoyancy, propulsive forces).

Equations of Motion - u, v, w

Definition of F_x, F_y, F_z, and **F**



Equations of Motion - u, v, w

Performing the indicated operations on the right hand side of equation (4) yields

$$\mathbf{F} = m\left(\frac{du}{dt} + qw - rv\right)\mathbf{i} + m\left(\frac{dv}{dt} + ru - pw\right)\mathbf{j} + m\left(\frac{dw}{dt} + pv - qu\right)\mathbf{k} \qquad (5)$$

Equations of Motion - u, v, w

Equation (5) can be rewritten in terms of its scalar components to yield the differential equations for u, v, and w. F_x, F_y, and F_z are the scalar components of the external forces vector.

$$\frac{du}{dt} = -qw + ru + \frac{F_x}{m} \qquad (6)$$

$$\frac{dv}{dt} = -ru + pw + \frac{F_y}{m} \qquad (7)$$

$$\frac{dw}{dt} = -pv + qu + \frac{F_z}{m} \qquad (8)$$

Equations of Motion - p, q, r

The angular momentum vector, **L**, about the origin of the body coordinate system (i.e., vehicle center of mass), is given by

$$\begin{aligned}\mathbf{L} &= \int_{Vol} \left[\mathbf{r}_m \times \left(\boldsymbol{\Omega} \times \mathbf{r}_m\right)\right] dm \\ &= \int_{Vol} \left[\boldsymbol{\Omega}\left(\mathbf{r}_m \bullet \mathbf{r}_m\right) - \mathbf{r}_m\left(\boldsymbol{\Omega} \bullet \mathbf{r}_m\right)\right] dm\end{aligned} \tag{9}$$

Where $\mathbf{r}_m$ is the position vector of an infinitesimal mass element dm

$$\mathbf{r}_m = x_m\mathbf{i} + y_m\mathbf{j} + z_m\mathbf{k} \tag{10}$$

Equations of Motion - p, q, r

Define the moments of inertia I_{xx}, I_{yy}, and I_{zz} as

$$\begin{aligned}\mathbf{I}_{xx} &= \int_{Vol} \left(y_m^2 + z_m^2\right) dm \\ \mathbf{I}_{yy} &= \int_{Vol} \left(x_m^2 + z_m^2\right) dm \\ \mathbf{I}_{zz} &= \int_{Vol} \left(x_m^2 + y_m^2\right) dm\end{aligned} \tag{11}$$

and the products of inertia as

$$\begin{aligned}\mathbf{I}_{xy} &= \int_{Vol} x_m y_m \, dm \\ \mathbf{I}_{xz} &= \int_{Vol} x_m z_m \, dm \\ \mathbf{I}_{yz} &= \int_{Vol} y_m z_m \, dm\end{aligned} \tag{12}$$

Beware that a different sign convention is sometimes used for the products of inertia. Also note that the moments and products of inertia as defined here are about the vehicle's center of mass (i.e., the body coordinate system origin).

Equations of Motion - p, q, r

Performing the integration described in equation (9) and using the definitions of the moments and products of inertia in equations (11) and (12) allows the angular momentum vector to be written as

$$\mathbf{L} = \left(I_{xx}p - I_{xy}q - I_{xz}r\right)\mathbf{i} + \left(-I_{xy}p + I_{yy}q - I_{yz}r\right)\mathbf{j} + \left(-I_{xz}p - I_{yz}q + I_{zz}r\right)\mathbf{k} \tag{13}$$

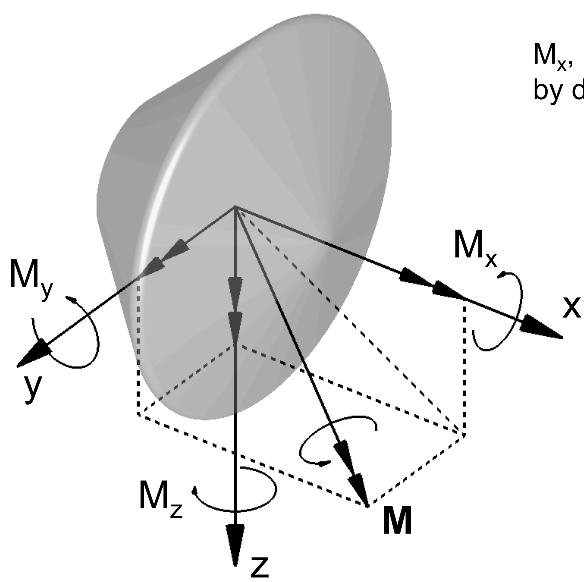
The (p, q, r) equations of motion can be obtained from

$$\mathbf{M} = \frac{d\mathbf{L}}{dt} + \mathbf{\Omega} \times \mathbf{L} \tag{14}$$

Where **M** is the external moments vector (e.g., gravitational, aerodynamic, buoyancy, propulsive moments) about the body coordinate system origin (i.e., vehicle center of mass). Notice the additional term **Ω** x **L**. Again, this term arises because the body coordinate system is rotating.

Equations of Motion - p, q, r

Definition of M_x, M_y, M_z, and **M**



M_x, M_y, M_z, and **M** are shown by double arrow <u>and</u> arc arrow

Note: In the literature, the components of the external moments vector, (M_x, M_y, M_z), are usually denoted by (l, m, n) or (L, M, N).

Equations of Motion - p, q, r

Performing the indicated operations on the right hand side of equation (14) yields

$$\mathbf{M} = \begin{bmatrix} I_{xx}\dfrac{dp}{dt} - I_{xy}\dfrac{dq}{dt} - I_{xz}\dfrac{dr}{dt} \\ -q\left(I_{xz}p + I_{yz}q - I_{zz}r\right) \\ -r\left(-I_{xy}p + I_{yy}q - I_{yz}r\right) \end{bmatrix}\mathbf{i} + \begin{bmatrix} -I_{xy}\dfrac{dp}{dt} + I_{yy}\dfrac{dq}{dt} - I_{yz}\dfrac{dr}{dt} \\ -p\left(-I_{xz}p - I_{yz}q + I_{zz}r\right) \\ -r\left(-I_{xx}p + I_{xy}q + I_{xz}r\right) \end{bmatrix}\mathbf{j}$$

$$+ \begin{bmatrix} -I_{xz}\dfrac{dp}{dt} - I_{yz}\dfrac{dq}{dt} + I_{zz}\dfrac{dr}{dt} \\ -p\left(I_{xy}p - I_{yy}q + I_{yz}r\right) \\ -q\left(I_{xx}p - I_{xy}q - I_{xz}r\right) \end{bmatrix}\mathbf{k} \qquad (15)$$

Equations of Motion - p, q, r

Equation (15) can be rewritten in terms of its scalar components to yield the differential equations for p, q, and r. M_x, M_y, and M_z are the scalar components of the external moments vector. These are often known as Euler's Equations of Motion.

$$I_{xx}\frac{dp}{dt} - I_{xy}\frac{dq}{dt} - I_{xz}\frac{dr}{dt} = q\left(I_{xz}p + I_{yz}q - I_{zz}r\right) + r\left(-I_{xy}p + I_{yy}q - I_{yz}r\right) + M_x \qquad (16)$$

$$-I_{xy}\frac{dp}{dt} + I_{yy}\frac{dq}{dt} - I_{yz}\frac{dr}{dt} = p\left(-I_{xz}p - I_{yz}q + I_{zz}r\right) + r\left(-I_{xx}p + I_{xy}q + I_{xz}r\right) + M_y \qquad (17)$$

$$-I_{xz}\frac{dp}{dt} - I_{yz}\frac{dq}{dt} + I_{zz}\frac{dr}{dt} = p\left(I_{xy}p - I_{yy}q + I_{yz}r\right) + q\left(I_{xx}p - I_{xy}q - I_{xz}r\right) + M_z \qquad (18)$$

Equations of Motion - ψ, θ, ϕ

The kinematic relationships between the derivatives of (ψ, θ, ϕ), and the components of the rotation rate vector, (p, q, r), are given by

$$d\psi/dt = \left(q\sin\phi + r\cos\phi\right)\sec\theta \qquad (19)$$

$$d\theta/dt = q\cos\phi - r\sin\phi \qquad (20)$$

$$d\phi/dt = p + \left(q\sin\phi + r\cos\phi\right)\tan\theta \qquad (21)$$

The derivation of these equations is too extensive to be presented here - see appendix A.

Note the difficulties that arise with equations (19) and (21) when $\theta = \pm\pi/2$.

Equations of Motion - X_P, Y_P, Z_P

The kinematic relationships between the derivatives of (X_P,Y_P, Z_P), and the components of the velocity vector, (u, v, w), are given in matrix form by

$$\begin{Bmatrix} dX_P/dt \\ dY_P/dt \\ dZ_P/dt \end{Bmatrix} = \left[T^{BP}\right]\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \qquad (22)$$

Where [T^{BP}] is a transformation matrix from the body to the planet coordinate system and the vehicle-carried planet coordinate system. This matrix is a function of the Euler Angles (ψ, θ, ϕ) and is given in the next slide. The derivation of this matrix is too extensive to be presented here - see appendix B.

Note that [T^{BP}] is orthogonal, and thus its inverse is equal to its transpose: $[T^{BP}]^{-1} = [T^{BP}]^T$.

Equations of Motion - X_P, Y_P, Z_P

$$\left[T^{BP}\right] = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \quad (23)$$

Euler Parameters / Quaternions

As has been noted, Euler Angles have problems with the singularity at $\theta = \pm\pi/2$. These problems can be avoided by using Euler Parameters (aka quaternions): ε_1, ε_2, ε_3, ε_4. Euler Parameters replace Euler Angles for describing the orientation of the body coordinate system with respect to the planet coordinate system and the vehicle-carried planet coordinate system. The four Euler Parameters become new state variables. With this replacement there are 13 equations of motion to integrate instead of 12.

A drawback of Euler Parameters is that the attitude of the vehicle is not obvious from the values of ε_1, ε_2, ε_3, and ε_4. This drawback can be overcome by integrating the equations of motion using Euler Parameters, but then reporting the orientation of the vehicle in terms of Euler Angles - which are calculated from the Euler Parameters.

For more on Euler Angles and Euler Parameters see appendix E of reference 1 (be aware of slightly different notation used in this reference).

Euler Parameters / Quaternions

$$\frac{d\varepsilon_1}{dt} = \frac{\varepsilon_4 p - \varepsilon_3 q + \varepsilon_2 r}{2} \qquad (24)$$

$$\frac{d\varepsilon_2}{dt} = \frac{\varepsilon_3 p + \varepsilon_4 q - \varepsilon_1 r}{2} \qquad (25)$$

$$\frac{d\varepsilon_3}{dt} = \frac{-\varepsilon_2 p + \varepsilon_1 q + \varepsilon_4 r}{2} \qquad (26)$$

$$\frac{d\varepsilon_4}{dt} = \frac{-\varepsilon_1 p - \varepsilon_2 q - \varepsilon_3 r}{2} \qquad (27)$$

Euler Parameters / Quaternions

The transformation matrix $[T^{BP}]$ can be written in terms of the Euler Parameters as

$$\left[T^{BP}\right] = \begin{bmatrix} \varepsilon_1^2 - \varepsilon_2^2 - \varepsilon_3^2 + \varepsilon_4^2 & 2(\varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4) & 2(\varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_2 + \varepsilon_3\varepsilon_4) & -\varepsilon_1^2 + \varepsilon_2^2 - \varepsilon_3^2 + \varepsilon_4^2 & 2(\varepsilon_2\varepsilon_3 - \varepsilon_1\varepsilon_4) \\ 2(\varepsilon_1\varepsilon_3 - \varepsilon_2\varepsilon_4) & 2(\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_4) & -\varepsilon_1^2 - \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 \end{bmatrix} \qquad (28)$$

Euler Parameters / Quaternions

Once the transformation matrix [T^{BP}] is known, by calculating it from equation (28), the Euler Angles can be determined from

$$\psi = \arctan\left(T_{21}^{BP} / T_{11}^{BP}\right) \qquad -\pi < \psi \le \pi \qquad (29)$$

$$\theta = \arcsin\left(-T_{31}^{BP}\right) \qquad -\pi/2 \le \theta \le \pi/2 \qquad (30)$$

$$\phi = \arctan\left(T_{32}^{BP} / T_{33}^{BP}\right) \qquad -\pi < \phi \le \pi \qquad (31)$$

In programming the equations for ψ and ϕ, a version of arctan that determines the correct quadrant for the angle must be used. This arctan function is usually known as "ATAN2".

Euler Parameters / Quaternions

From the transformation matrix [T^{BP}] the Euler Parameters can be determined from

$$tr\left[T^{BP}\right] = T_{11}^{BP} + T_{22}^{BP} + T_{33}^{BP} \qquad (32)$$

$$\varepsilon_4 = \sqrt{\left(tr\left[T^{BP}\right] + 1\right)/4} \qquad (33)$$

$$\varepsilon_1 = \left(T_{32}^{BP} - T_{23}^{BP}\right)/4\varepsilon_4 \qquad (34)$$

$$\varepsilon_2 = \left(T_{13}^{BP} - T_{31}^{BP}\right)/4\varepsilon_4 \qquad (35)$$

$$\varepsilon_3 = \left(T_{21}^{BP} - T_{12}^{BP}\right)/4\varepsilon_4 \qquad (36)$$

Euler Parameters / Quaternions

The Euler Parameters satisfy the relationship

$$\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2 = 1 \qquad (37)$$

Because of roundoff error during numerical integration of equations (24) to (27), the relationship shown in equation (37) may cease to be met. The Euler Parameters then need to be renormalized. One approach is to multiply them by the constant K_ε

$$K_\varepsilon = 1 \Big/ \sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2} \qquad (38)$$

Thus, the renormalized values of the Euler Parameters are

$$\varepsilon_{iR} = K_\varepsilon \varepsilon_i \quad \text{for } i = 4 \qquad (39)$$

In appendix E of reference 1, a different way of renormalizing the Euler Parameters is presented.

External Forces and Moments

External forces and moments can arise from several sources, including: gravitational, aerodynamic, buoyancy, and propulsive.

In this set of lecture notes only two of these will be considered

- Gravitational. Described by the force and moment vectors, $\mathbf{F}_g$ and $\mathbf{M}_g$, and their scalar components.
- Aerodynamic. Described by the force and moment vectors, $\mathbf{F}_A$ and $\mathbf{M}_A$, and their scalar components.

External Forces and Moments

Thus, for our purposes

$$\mathbf{F} = \mathbf{F}_g + \mathbf{F}_A \quad (40)$$

$$F_x = F_{g,x} + F_{A,x} \quad (41)$$

$$F_y = F_{g,y} + F_{A,y} \quad (42)$$

$$F_z = F_{g,z} + F_{A,z} \quad (43)$$

$$\mathbf{M} = \mathbf{M}_g + \mathbf{M}_A \quad (44)$$

$$M_x = M_{g,x} + M_{A,x} \quad (45)$$

$$M_y = M_{g,y} + M_{A,y} \quad (46)$$

$$M_z = M_{g,z} + M_{A,z} \quad (47)$$

Note: all components of vectors on this slide are in the body coordinate system.

External Forces and Moments - Gravity

Let W be the weight of the vehicle

$$W = mg \quad (48)$$

The gravity force vector can be expressed in terms of the planet coordinate system as

$$\mathbf{F}_g = (0)\mathbf{i}_P + (0)\mathbf{j}_P + W\mathbf{k}_P \quad (49)$$

This vector needs to be expressed in terms of the body coordinate system. To do this the transformation matrix $[T^{BP}]^T$ is used.

External Forces and Moments - Gravity

$$\begin{Bmatrix} F_{g,x} \\ F_{g,y} \\ F_{g,z} \end{Bmatrix} = \left[T^{BP}\right]^T \begin{Bmatrix} 0 \\ 0 \\ W \end{Bmatrix} \tag{50}$$

Using the expressions for $[T^{BP}]$ in equations (23) and (28), the scalar components of the gravity vector in the body coordinate system can be written as

External Forces and Moments - Gravity

$$\begin{aligned} F_{g,x} &= -W\sin\theta \\ &= 2W\left(\varepsilon_1\varepsilon_3 - \varepsilon_2\varepsilon_4\right) \end{aligned} \tag{51}$$

$$\begin{aligned} F_{g,y} &= W\sin\phi\cos\theta \\ &= 2W\left(\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_4\right) \end{aligned} \tag{52}$$

$$\begin{aligned} F_{g,z} &= W\cos\phi\cos\theta \\ &= W\left(-\varepsilon_1^2 - \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2\right) \end{aligned} \tag{53}$$

External Forces and Moments - Gravity

Notice that since we have defined the origin of the body coordinate system to be at the center of mass of the vehicle, the moment vector due to gravity, $\mathbf{M}_g$, (and thus all its component) in the body coordinate system are zero.

$$\mathbf{M}_g = 0 \tag{54}$$

$$M_{g,x} = 0 \tag{55}$$

$$M_{g,y} = 0 \tag{56}$$

$$M_{g,z} = 0 \tag{57}$$

External Forces and Moments - Aero

Modeling the aerodynamic forces and moments is difficult. The end result is just that - a model.

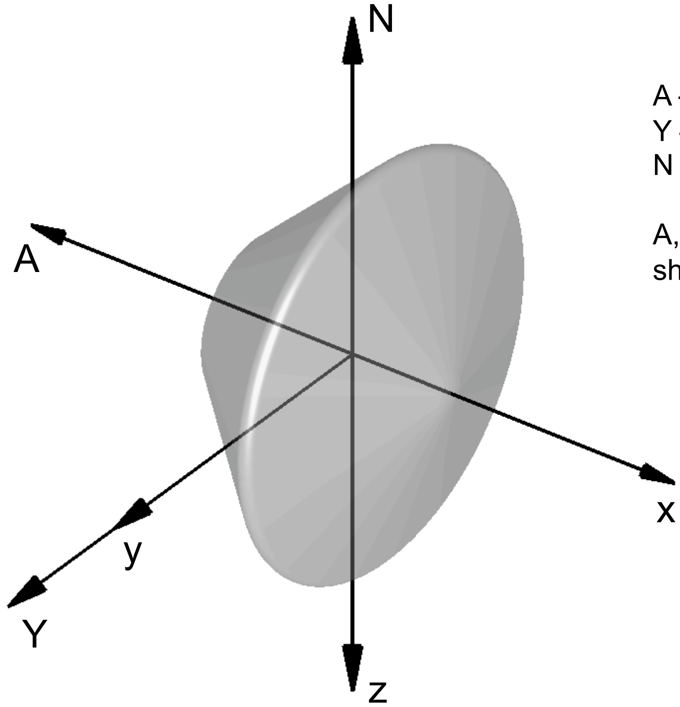
In this set of lecture notes the aerodynamic forces and moments are modeled as follows

$$F_{A,x} = \text{-Axial Force} = -A = -q_\infty S C_A \tag{58}$$

$$F_{A,y} = \text{Side Force} = Y = q_\infty S C_Y \tag{59}$$

$$F_{A,z} = \text{-Normal Force} = -N = -q_\infty S C_N \tag{60}$$

External Forces and Moments - Aero



A – axial force
Y – side force
N – normal force

A, Y, and N, as shown, are positive

External Forces and Moments - Aero

$$M_{A,x} = \text{Roll Moment} = q_\infty S D C_l + q_\infty S D C_{l_p}\left[p\left(\frac{D}{2V_\infty}\right)\right] \tag{61}$$

$$M_{A,y} = \text{Pitch Moment} = q_\infty S D C_m + q_\infty S D C_{m_{\left(q + \frac{d\alpha}{dt}\right)}}\left[\left(\frac{q + d\alpha/dt}{2}\right)\left(\frac{D}{2V_\infty}\right)\right] \tag{62}$$

$$M_{A,z} = \text{Yaw Moment} = q_\infty S D C_n + q_\infty S D C_{n_{\left(r - \frac{d\beta}{dt}\right)}}\left[\left(\frac{r - d\beta/dt}{2}\right)\left(\frac{D}{2V_\infty}\right)\right] \tag{63}$$

External Forces and Moments - Aero

Where,

$$V_\infty = \sqrt{u^2 + v^2 + w^2} \qquad (64)$$

(airspeed)

$$q_\infty = \tfrac{1}{2}\rho V_\infty^2 \qquad (65)$$

(dynamic pressure)

Note that V_∞ can be defined as shown in equation (64) because we have assumed no wind, and thus the velocity components (u, v, w) are the same as the airspeed components (u_∞, v_∞, w_∞). If there is wind the velocity and airspeed components will not be the same! The wind will need to be taken into account to determine the airspeed components and the airspeed. Also, remember that the atmospheric density, ρ, is a function of the altitude, $h = -Z_P$.

External Forces and Moments - Aero

Comments on Equations (58) to (63)

- The reference area for all aerodynamic coefficients is S.
- The reference length for all aerodynamic coefficients is D.
- The approach used here is to define the static aerodynamic coefficients C_A and C_N in the body coordinate system as is usually done for entry vehicles. In aircraft applications it is common to use the equivalent static aerodynamic coefficients C_L and C_D (lift and drag, respectively). The transformation between (C_A, C_N) and (C_L, C_D) is a simple one involving the angle of attack, α. I leave it up to you to derive it.
- It is assumed here that the aerodynamic coefficients can only be functions of the vehicle geometry, location of the vehicle center of mass, the instantaneous value of the state variables and their derivatives (which include the angle of attack, angle of sideslip, and their derivatives), the Mach number, the Reynolds number, and the Knudsen number as appropriate.

External Forces and Moments - Aero

- C_A, C_Y, C_N, C_l, C_m, C_n are static aerodynamic coefficients because they contribute forces and moments that are not functions of rate-dependent quantities such as (p, q, r) and ($d\alpha/dt$ and $d\beta/dt$).
- The moment aerodynamic coefficients, both static (C_l, C_m, C_n) and dynamic (C_{lp}, $C_{m(q+d\alpha/dt)}$, $C_{n(r-d\beta/dt)}$), depend on the location of the vehicle's center of mass. The values of the static moment aerodynamic coefficients can be easily shifted from one center of mass location to another. However, the dynamic moment aerodynamic coefficients usually cannot be shifted.
- The nondimensionalizing term $D/(2V_\infty)$ in equations (61) to (63) is not universal. Using D/V_∞ is also common. Sometimes different reference lengths are used for each one of these equations. Be careful when using and interpreting dynamic moment aerodynamic coefficients - be sure you are clear as to the nondimensionalization scheme.

External Forces and Moments - Aero

- In practice it is very difficult to separate the q and $d\alpha/dt$ portions of the dynamic pitching moment coefficients. Similarly for r and $d\beta/dt$. Thus, they are often used in combination as shown in equations (62) and (63).
- Notice the sign difference in the term ($q + d\alpha/dt$) and ($r - d\beta/dt$) in equations (62) and (63). These differences are due to the definitions of α and β. This point will be discussed later in the relative wind angles section of these lecture notes.
- When using legacy aerodynamic coefficient data be clear as to how they were nondimensionalized, the assumed reference areas and lengths, and the assumed units (e.g., radians or degrees?).

Relative Wind Angles

The static and dynamic aerodynamic coefficients are usually specified as functions of the angle of attack, α, and the angle of sideslip, β

$$\alpha = \arctan\left(\frac{w}{u}\right) \qquad -\pi < \alpha \le \pi \tag{66}$$

$$\beta = \arcsin\left(\frac{v}{V_\infty}\right) \qquad -\frac{\pi}{2} < \beta \le \frac{\pi}{2} \tag{67}$$

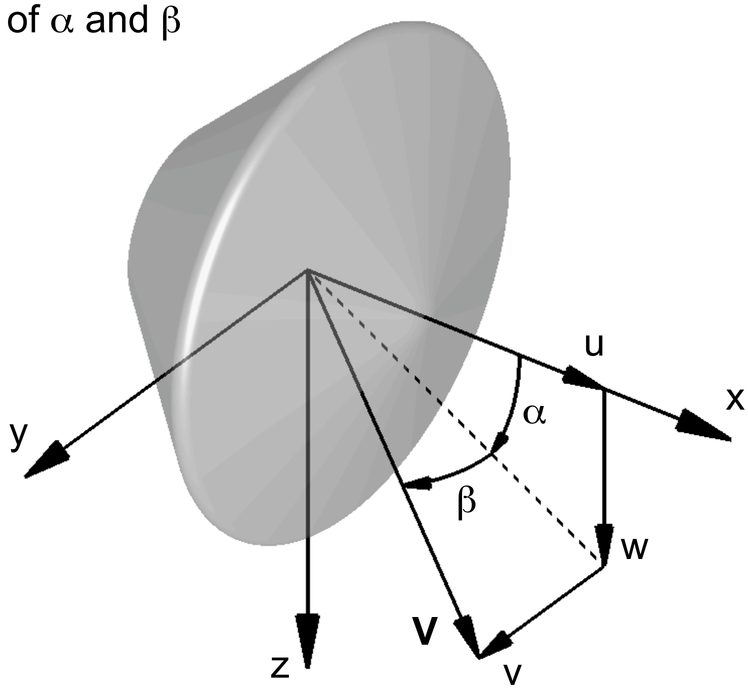
Relative Wind Angles

The following should be noted regarding α and β:

- α and β are functions of the state variables
- Only two angles, α and β, are required to fully specify the direction of the relative wind.
- The definitions of α and β given in equations (66) and (67) are standardized in the literature.
- If u and w are zero, α is not defined. If V_∞ is zero, β is not defined.
- In programming the equation for α, a version of arctan that determines the correct quadrant for the angle must be used. This arctan function is usually known as “ATAN2”.
- The definitions for α and β are not symmetric. Note the difference in the inverse trigonometric function used, and the differences in the domains of α and β.
- Note that α and β can be defined as shown in equations (66) and (67) using (u, v, w) because we have assumed no wind, and thus the velocity components (u, v, w) are the same as the airspeed components (u_∞, v_∞, w_∞). If there is wind the velocity and airspeed components will not be the same! The wind will need to be taken into account to determine the airspeed components, and α and β.
- If (u, v, w) are known, then (α, β, V_∞) can be determined. Conversely, if (α, β, V_∞) are known, (u, v, w) can be determined.

Relative Wind Angles

Definition of α and β



Relative Wind Angles

The relative wind direction can also be specified in terms of the total angle of attack, α_T, and the total angle of attack clock angle, $\phi_{\alpha T}$

$$\alpha_T = \arccos\left(\frac{u}{V_\infty}\right) \qquad 0 \le \alpha_T \le \pi \qquad (68)$$

$$\phi_{\alpha_T} = \arctan\left(\frac{v}{w}\right) \qquad -\pi < \phi_{\alpha_T} \le \pi \qquad (69)$$

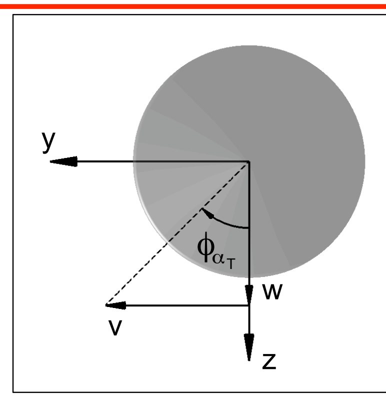
Relative Wind Angles

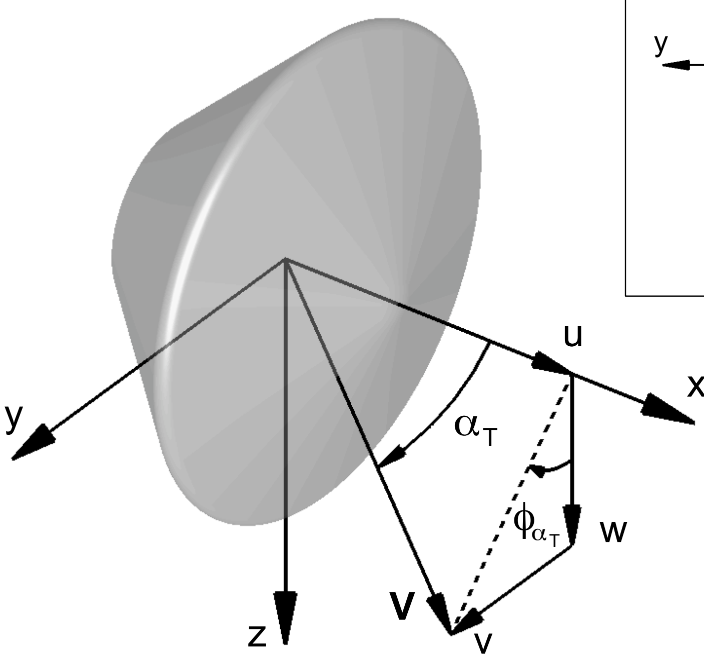
The following should be noted regarding α_T and $\phi_{\alpha T}$:

- α_T and $\phi_{\alpha T}$ are functions of the state variables
- Only two angles, α_T and $\phi_{\alpha T}$, are required to fully specify the direction of the relative wind.
- The definition of α_T given in equation (68) is standardized in the literature. However, the definition of $\phi_{\alpha T}$ is not. When reading the literature and/or analyzing data make sure you understand how $\phi_{\alpha T}$ is defined.
- If V_∞ is zero, α_T is not defined. If v and w are zero, $\phi_{\alpha T}$ is not defined.
- In programming the equation for $\phi_{\alpha T}$, a version of arctan that determines the correct quadrant for the angle must be used. This arctan function is usually known as "ATAN2".
- Note that α_T and $\phi_{\alpha T}$ can be defined as shown in equations (68) and (69) using (u, v, w) because we have assumed no wind, and thus the velocity components (u, v, w) are the same as the airspeed components (u_∞, v_∞, w_∞). If there is wind the velocity and airspeed components will not be the same! The wind will need to be taken into account to determine the airspeed components, and α and β.
- If (u, v, w) are known, then (α_T, $\phi_{\alpha T}$, V_∞) can be determined. Conversely, if (α_T, $\phi_{\alpha T}$, V_∞) are known, (u, v, w) can be determined.

Relative Wind Angles

Definition of α_T and $\phi_{\alpha T}$





Relative Wind Angles

The relative wind direction can be specified in terms of either (α, β) or $(\alpha_T, \phi_{\alpha T})$. We can go from one set to the other by using the following equations

$$\alpha = \arctan\left(\frac{\sin\alpha_T \cos\phi_{\alpha_T}}{\cos\alpha_T}\right) \qquad -\pi < \alpha \leq \pi \qquad (70)$$

$$\beta = \arcsin\left(\sin\alpha_T \cos\phi_{\alpha_T}\right) \qquad -\pi/2 \leq \beta \leq \pi/2 \qquad (71)$$

$$\alpha_T = \arccos(\cos\alpha\cos\beta) \qquad 0 \leq \alpha_T \leq \pi \qquad (72)$$

$$\phi_{\alpha_T} = \arctan\left(\frac{\sin\beta}{\sin\alpha\cos\beta}\right) \qquad -\pi < \phi_{\alpha_T} \leq \pi \qquad (73)$$

Relative Wind Angles

The following should be noted regarding equations (70) to (73):

- Only one set of two angles, (α, β) or $(\alpha_T, \phi_{\alpha T})$, is required to fully specify the direction of the relative wind. Once one set is known, the other one can be calculated.
- In programming equations (70) and (73), a version of arctan that determines the correct quadrant for the angle must be used. This arctan function is usually known as “ATAN2”.
- Do not simplify the terms inside the inverse tangent function. For example, in equation (70) do not simplify

  $$[(\sin\alpha_T\cos\phi_{\alpha T})/\cos\alpha_T]$$

  into

  $$\tan\alpha_T\cos\phi_{\alpha T}$$

  Information regarding the quadrant for the angle will be lost.
- Earlier comments regarding α, β, α_T, and $\phi_{\alpha T}$ are still valid, including those remarks regarding situations when the various angles are not defined.

Relative Wind Angles

In some aerodynamic model formulations the derivatives of the relative wind angles with respect to time will be needed. These derivatives can be determined directly from the state variables as shown below. They are derived from equations (66) to (69) by the use of calculus, algebra, and some trigonometric identities; I leave it up to you to verify their derivation.

Relative Wind Angles

$$\frac{d\alpha}{dt} = \frac{u(dw/dt) - (du/dt)w}{u^2 + w^2} \tag{74}$$

$$\frac{d\beta}{dt} = \frac{(dv/dt)V_\infty^2 - v\left[u(du/dt) + v(dv/dt) + w(dw/dt)\right]}{V_\infty^2\sqrt{u^2 + w^2}} \tag{75}$$

$$\frac{d\alpha_T}{dt} = \frac{u\left[u(du/dt) + v(dv/dt) + w(dw/dt)\right] - (du/dt)V_\infty^2}{V_\infty^2\sqrt{v^2 + w^2}} \tag{76}$$

$$\frac{d\phi_{\alpha_T}}{dt} = \frac{(dv/dt)w - v(dw/dt)}{v^2 + w^2} \tag{77}$$

Relative Wind Angles

The following should be noted regarding equations (74) to (77):

- The angular units of these equations are radians. Since we usually use seconds as the time units, the units of the derivatives are radians/second.
- Again, note that in the following equations (u, v, w) and derived quantities are used. We can do this because we have assumed no wind. In the presence of wind (u, v, w) and quantities calculated from them need to be replaced by the airspeed components (u_∞, v_∞, w_∞), respectively.
- Sometimes these derivatives are calculated by finite differencing within computer codes. When wind is not zero, finite differencing may be the only way to obtain these derivatives.
- There are cases for which these derivatives are not defined.

Example Aerodynamic Model

To define an example aerodynamic model let us consider a blunt-body entry vehicle. We will assume the following:

- The vehicle is traveling at supersonic speeds. In the flight regime of interest for this model the aerodynamic coefficients are often insensitive to Mach number. Here we make the approximation that they are independent of Mach number.
- The vehicle is geometrically axisymmetric, but the center of mass is not on the axis of symmetry.
- Data are available for the static and dynamic aerodynamic coefficients.
- The reference length for the aerodynamic data is the vehicle's maximum diameter, D. The reference length for the aerodynamic data is based on D: $S = \pi(D/2)^2$.

Example Aerodynamic Model

Static Aerodynamic Coefficients

- The model coordinate system has axes (ξ, η, ζ). The ξ axis is coincident with the axis of symmetry of the entry vehicle.
- The (ξ, η, ζ) axes are parallel to the body coordinate system axes (x, y, z), respectively.
- The center of mass of the vehicle (i.e., the origin of the body coordinate system) is located at (ξ_{CM}, η_{CM}, ζ_{CM}) in the model coordinate system. The vector from the origin of the model coordinate system to the origin of the body coordinate system is $\mathbf{r}_{CM}$.
- Because the vehicle is axisymmetric, the static aerodynamic coefficients are given as functions of the total angle of attack, α_T, only, in terms of axisymmetric aerodynamic coefficients:

$$C_{AT}(\alpha_T), \quad C_{NT}(\alpha_T), \quad C_{lOT}(\alpha_T), \quad C_{mOT}(\alpha_T)$$

Example Aerodynamic Model

Static Aerodynamic Coefficients - Continued

- Because the vehicle is axisymmetric, the aerodynamic forces vector components specified by C_{AT} and C_{NT} lie in the plane specified by the ξ axis and the velocity vector **V**.*
- Because the vehicle is axisymmetric, the aerodynamic moment vector component specified by $C_{mOT}(\alpha_T)$ is perpendicular to the plane specified by the ξ axis and the velocity vector **V**.* Also note that this moment is about the origin, O, of the (ξ, η, ζ) coordinate system, not about the origin of the body coordinate system (x, y, z) which is centered on the vehicle's center of mass.
- Because the vehicle is axisymmetric, the roll moment coefficient about the ξ axis, C_{lOT}, is zero.

*Given our assumption of no wind, the velocity vector, **V**, is the same as the airspeed vector, $\mathbf{V}_\infty$. When there is wind this will not generally be the case. It is the airspeed velocity vector that is needed to determine the relative wind angles, airspeed, and dynamic pressure.

Example Aerodynamic Model

Static Aerodynamic Coefficients - Continued

- Because the vehicle is axisymmetric, $C_{NT} = 0$ and $C_{mOT} = 0$ when $\alpha_T = 0$.
- The aerodynamic forces associated with axisymmetric static aerodynamic coefficients are

$$A_T = q_\infty S C_{AT} \tag{78}$$

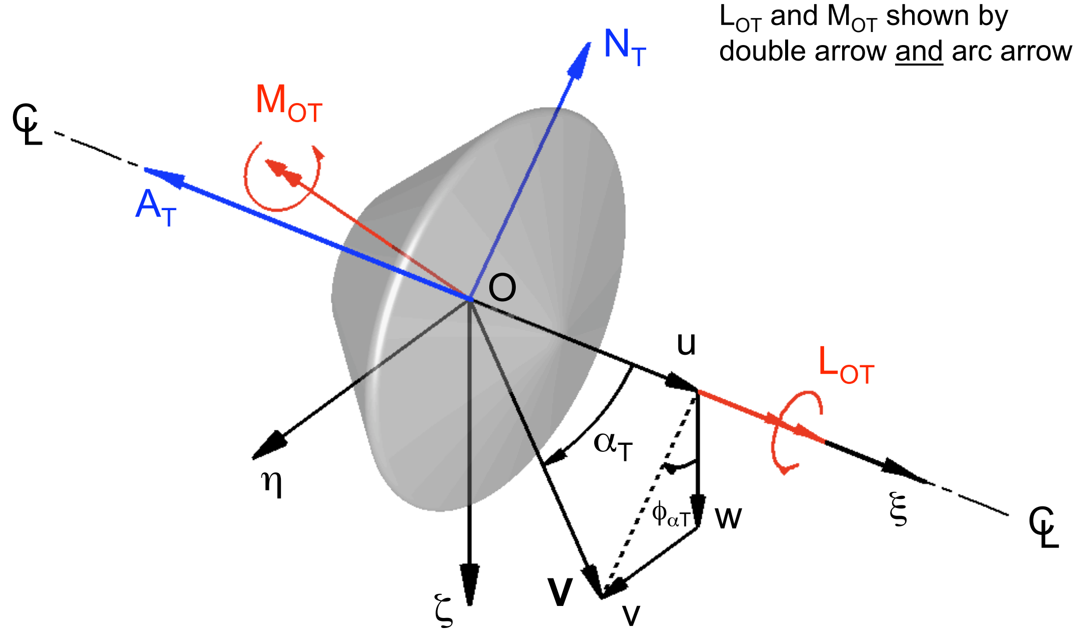
$$N_T = q_\infty S C_{NT} \tag{79}$$

$$L_{OT} = q_\infty S D C_{lOT} \tag{80}$$

$$M_{OT} = q_\infty S D C_{mOT} \tag{81}$$

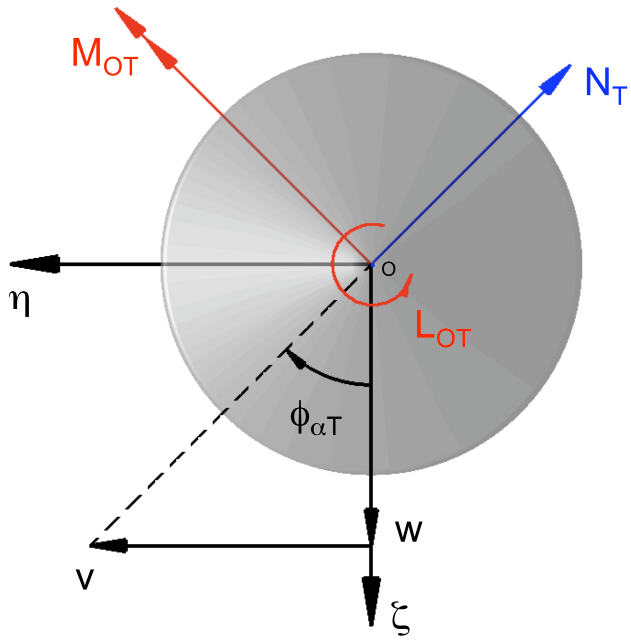
Example Aerodynamic Model

L_{OT} and M_{OT} shown by double arrow <u>and</u> arc arrow



Example Aerodynamic Model

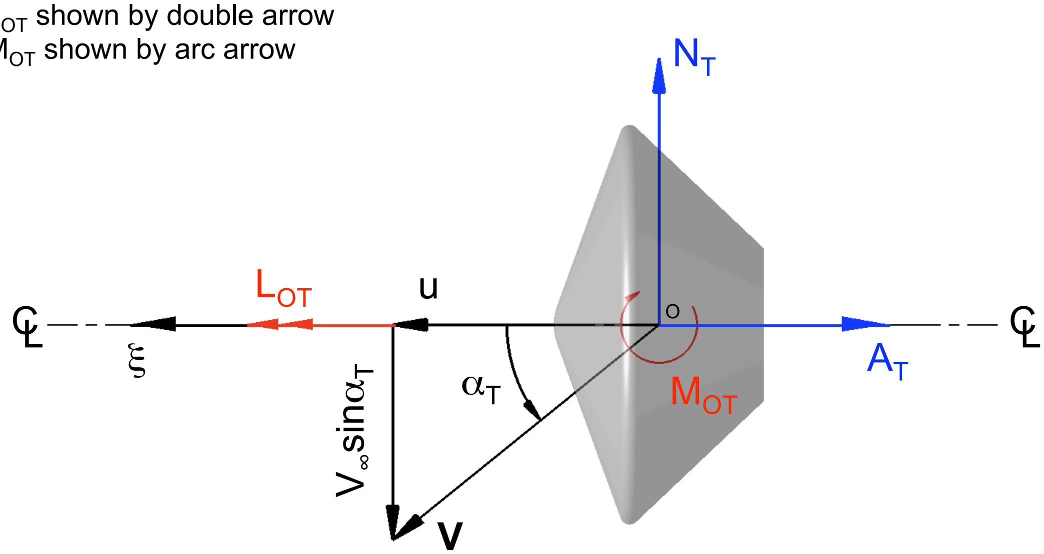
L_{OT} shown by arc arrow
M_{OT} shown by double arrow



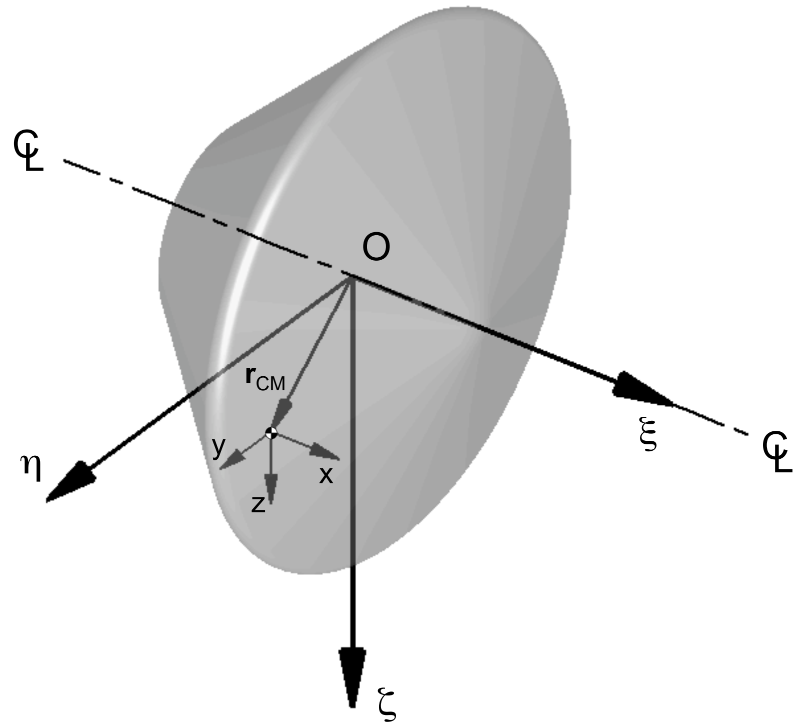
Example Aerodynamic Model

ξ and **V** on plane of slide (α_T plane)
η not necessarily perpendicular to plane of slide

L_{OT} shown by double arrow
M_{OT} shown by arc arrow



Example Aerodynamic Model



Example Aerodynamic Model

Static Aerodynamic Coefficients - Continued

The static aerodynamic coefficients, C_A, C_Y, C_N, C_l, C_m, and C_n, in the body coordinate system can be calculated from the following equations. Note that the roll, pitch, and yaw moments must be transferred to the body coordinate system origin (i.e., the vehicle's center of mass). The derivation of these equations is presented in appendix C.

$$C_A = C_{A_T} \tag{82}$$

$$C_Y = -C_{N_T} \sin\phi_{\alpha_T} \tag{83}$$

$$C_N = C_{N_T} \cos\phi_{\alpha_T} \tag{84}$$

Example Aerodynamic Model

Static Aerodynamic Coefficients - Continued

Assumed zero per earlier discussion

$$C_l = \cancel{C_{l_{OT}}} + C_N \frac{\eta_{CM}}{D} + C_Y \frac{\zeta_{CM}}{D} \quad (85)$$

$$C_m = C_{m_{OT}} \cos\phi_{\alpha_T} - C_N \frac{\xi_{CM}}{D} + C_A \frac{\zeta_{CM}}{D} \quad (86)$$

$$C_n = -C_{m_{OT}} \sin\phi_{\alpha_T} - C_Y \frac{\xi_{CM}}{D} - C_A \frac{\eta_{CM}}{D} \quad (87)$$

Example Aerodynamic Model

Some comments regarding the static aerodynamic coefficients just presented.

- The formulation presented for the static aerodynamic coefficients, C_A, C_Y, C_N, C_l, C_m, and C_n, in equations (82) to (87) are in terms of the total angle of attack, α_T, and the total angle of attack clock angle, $\phi_{\alpha T}$.

- Even though the vehicle has been assumed to be axisymmetric, the roll moment coefficient about the center of mass, C_l, is not necessarily always zero if the vehicle's center of mass is not on the body's axis of symmetry.

Example Aerodynamic Model

Dynamic Aerodynamic Coefficients

- For this example we assume that the dynamic aerodynamic coefficients C_{lp}, $C_{m(q+d\alpha/dt)}$, and $C_{n(r-d\beta/dt)}$ have been determined from a ballistic range test with the correct center of mass location.
- $C_{m(q+d\alpha/dt)}$ is assumed to be a function of α.
- $C_{n(r-d\beta/dt)}$ is assumed to be a function of β.
- The functional forms of $C_{m(q+d\alpha/dt)}$ and $C_{n(r-d\beta/dt)}$ can depend greatly on the vehicle - no specific form is given here.
- Identifying appropriate values of C_{lp}, $C_{m(q+d\alpha/dt)}$, and $C_{n(r-d\beta/dt)}$, even when a lot of data available, is difficult.
- The values of the dynamic aerodynamic coefficients cannot be easily transferred to another center of mass location - usually a new set of tests is required.

Flight Path Angle

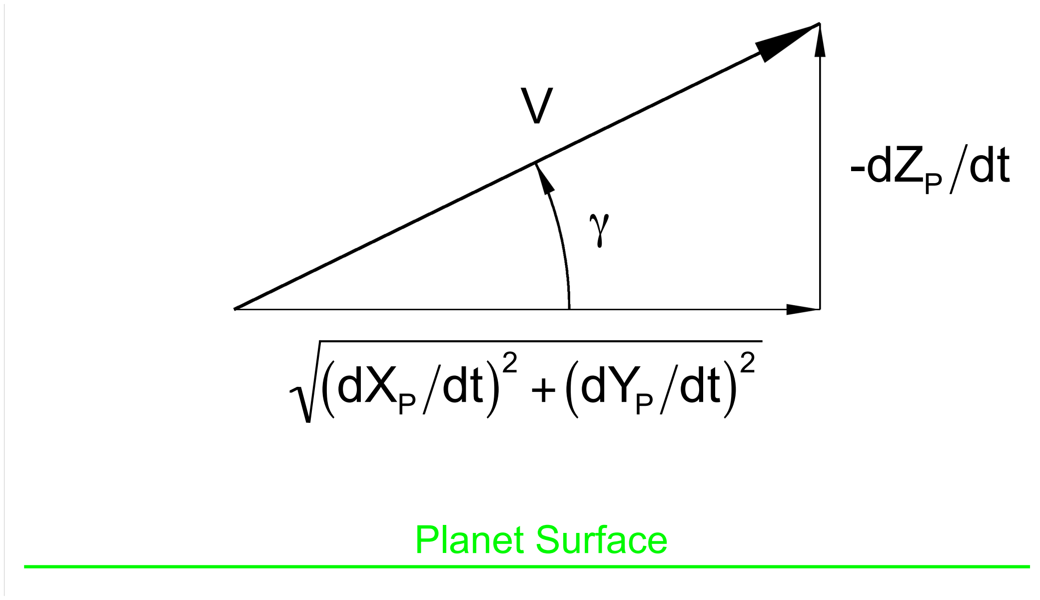
It is often of interest to know what is the orientation of the velocity vector, **V**, with respect to the local horizontal (i.e., surface of the planet). This orientation is often given in terms of the flight path angle, γ, defined by

$$\gamma = \arctan\left[\frac{-(dZ_P/dt)}{\sqrt{(dX_P/dt)^2 + (dY_P/dt)^2}}\right] \qquad -\pi/2 < \gamma \le \pi/2 \qquad (88)$$

Note the following:

- The flight path angle only describes the orientation of the velocity vector, **V**, above ($\gamma > 0$) or below ($\gamma < 0$) the horizon. It does not include information regarding the direction in the (X_P, Y_P) plane (i.e., north, east, south, west).
- The opposite sign convention, $\gamma > 0$ below the horizon, is also commonly used in the literature.
- Other quantities also called “flight path angle” can also be defined. Be clear as to which one is being used.
- In programming equation (88), a version of arctan that determines the correct quadrant for the angle must be used. This arctan function is usually known as “ATAN2”.

Flight Path Angle



Numerical Integration of the Equations of Motion

Ordinary differential equations (ODEs) can be divided into two groups:

Explicit ODEs: $dX/dt = f_e(X, t)$

Implicit ODEs: $f_i(dX/dt, X, t) = 0$

Although all explicit ODEs can be written in Implicit form, not all implicit ODEs can be written in Explicit form.

The equations above can stand for a set of coupled ODEs such as those discussed in this lecture.

Note that the description "Explicit" and "Implicit" are used here to describe the nature of the differential equations - not the numerical algorithm used to integrate them. These terms are used with a different meaning to describe numerical integration algorithms - this second sense is not used in these lecture notes.

Different numerical algorithms are used to integrate explicit and implicit ODEs.

Numerical Integration of the Equations of Motion

For the types of flight mechanics problems discussed in this lecture…

Case 1

<u>IF</u> the external force and moment vectors depend only on the state variables, and do not depend on the time derivatives of the state variables, it is possible to write the equations of motion ODEs in explicit form.

Case 2

<u>IF</u> the external force and moment vectors depend on the state variables, and are linearly dependent on the time derivatives of the state variables, it is possible to write the equation of motion ODEs in explicit form.

Case 3

<u>IF</u> the external force and moment vectors are nonlinearly dependent on the time derivatives of the state variables, it may not be possible to write the equations of motion ODEs in explicit form.

Numerical Integration of the Equations of Motion

Note the following:

- In this set of lecture notes we are only considering gravity and aerodynamic forces and moments.
- The gravity force and moment vectors in equations (49) to (57) are only dependent on the state variables, they do not include any time derivatives of the state variables.
- The model described by equations (58) to (63) for the aerodynamic force and moment vectors depend on the state variables, and linearly on the time derivatives of the state variables through the terms $d\alpha/dt$ and $d\beta/dt$ in equations (62) and (63).

If we then assume that all the aerodynamic coefficients (static and dynamic) are only functions of the state variables, and not on the time derivatives of the state variables, then the equations of motion, including the force and moment vectors as modeled here, fall under Case 2 in the previous slide.

Numerical Integration of the Equations of Motion

Numerical Integration Algorithms

The most common numerical integration algorithms require that the ODEs be written in explicit form

$$dX/dt = f_e(X, t)$$

Runge-Kutta methods are common for solving explicit ODEs. The solver “ode45” in MATLAB uses this method.

More difficult to find are numerical integration algorithms for ODEs written in implicit form

$$f_i(dX/dt, X, t) = 0$$

The solver “ode15i” in MATLAB can integrate implicit ODEs.

Numerical Integration of the Equations of Motion

Where am going with all this?

If you are writing your own code to solve flight mechanics problems as posed here, you will need to decide in which form you are going to, or can, write the ODEs. Then, you will have to choose an appropriate numerical integration algorithm.

You can always write the ODEs in implicit form, and use a numerical integration algorithm intended for use with implicit ODEs.

Sometimes additional approximations are made. Specifically, the equations of motion ODEs will be written in “pseudo-explicit” form, with the external force and moment vectors on the right hand side of the equations, even though these vectors may depend on time derivatives of the state variables. A numerical integration algorithm for explicit ODEs will be used, and time derivatives of the state variables in the right hand side of the equations will be determined by finite differencing. Note that doing this adds an additional level of approximation in the numerical integration of the ODEs - you will have to decide whether this approximation yields acceptable results.

Numerical Integration of the Equations of Motion

Some remaining points…

Notice that equations (16) to (18) include all three derivatives dp/dt, dq/dt, and dr/dt in each of the equations. You can rewrite these equations to yield:

$$\begin{Bmatrix} dp/dt \\ dq/dt \\ dr/dt \end{Bmatrix} = [A]^{-1} \left[\begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} - [B] \begin{Bmatrix} p^2 \\ q^2 \\ r^2 \\ pq \\ pr \\ qr \end{Bmatrix} \right] \tag{89}$$

Numerical Integration of the Equations of Motion

where,

$$[A] = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \tag{90}$$

$$[B] = \begin{bmatrix} 0 & -I_{yz} & I_{yz} & -I_{xz} & I_{xy} & (I_{zz} - I_{yy}) \\ I_{xz} & 0 & -I_{xz} & I_{yz} & (I_{xx} - I_{zz}) & -I_{xy} \\ -I_{xy} & I_{xy} & 0 & (I_{yy} - I_{xx}) & -I_{yz} & I_{xz} \end{bmatrix} \tag{91}$$

I leave it up to you to re-derive these equations.

Note that equation (85) is not necessarily in explicit form. If the external moments contain time derivatives of the state variables (for example terms involving dα/dt and/or dβ/dt), equation (85) is not in explicit form.

Numerical Integration of the Equations of Motion

Initial Conditions

To integrate the equations of motion for a particular case, you will need the initial conditions.

If the ODEs are in explicit form, you will only need initial conditions for the state variables.

If the ODEs are in implicit form, you will need initial conditions for the state variables and their time derivatives. The initial conditions for the state variables time derivatives can be calculated from the ODEs - they depend only on the initial conditions for the state variables. However, this calculation may need to be performed numerically.

Symbols

A	axial force
A_T	axisymmetric axial force acting at the origin of the model coordinate system (O)
[A]	moment and product of inertia matrix
[B]	moment and product of inertia matrix
C_A	axial force coefficient
C_{AT}	axisymmetric axial force coefficient
C_D	drag coefficient
C_L	lift coefficient
C_l	roll moment coefficient
C_{lOT}	axisymmetric roll moment coefficient
C_{lp}	roll moment dynamic coefficient
C_m	pitch moment coefficient
C_{mOT}	axisymmetric pitch moment coefficient
$C_{m(q+d\alpha/dt)}$	pitch moment dynamic coefficient
C_N	normal force coefficient
C_{NT}	axisymmetric normal force coefficient
C_n	yaw moment coefficient
$C_{n(r-d\beta/dt)}$	pitch moment dynamic coefficient
C_Y	side force coefficient
D	reference length

Symbols

$\mathbf{F}$	external forces vector
$\mathbf{F}_A$	aerodynamic force vector
$F_{A,x}$, $F_{A,y}$, $F_{A,z}$	components of the aerodynamic force vector, $\mathbf{F}_A$, in the body coordinate system
$\mathbf{F}_g$	gravitational force vector
$F_{g,x}$, $F_{g,y}$, $F_{g,z}$	components of the gravitational force vector, $\mathbf{F}_g$, in the body coordinate system
F_x, F_y, F_z	components of the external forces vector, $\mathbf{F}$, in the body coordinate system
f_e, f_i	functions
g	acceleration of gravity
h	altitude
I_{xx}, I_{yy}, I_{zz}	moments of inertia about the vehicle's center of mass
I_{xy}, I_{xz}, I_{yz}	products of inertia about the vehicle's center of mass
$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	unit vectors for the body coordinate system
$\mathbf{i}_P$, $\mathbf{j}_P$, $\mathbf{k}_P$	unit vectors for the planet coordinate system
$\mathbf{i}_V$, $\mathbf{j}_V$, $\mathbf{k}_V$	unit vectors for the vehicle-carried planet coordinate system
K_ε	renormalizing factor for the Euler Parameters
$\mathbf{L}$	angular momentum vector
L_{OT}	axisymmetric roll moment acting at the origin of the model coordinate system

Symbols

$\mathbf{M}$	external moments vector about the body coordinate system origin (i.e., vehicle center of mass)
$\mathbf{M}_A$	aerodynamic moment vector about the body coordinate system origin (i.e., vehicle center of mass)
$M_{A,x}$, $M_{A,y}$, $M_{A,z}$	components of the aerodynamic moment vector, $\mathbf{M}_A$, in the body coordinate system
$\mathbf{M}_g$	gravitational moment vector about the body coordinate system origin (i.e., vehicle center of mass) – zero in the present discussion
$M_{g,x}$, $M_{g,y}$, $M_{g,z}$	components of the aerodynamic moment vector, $\mathbf{M}_g$, in the body coordinate system – zero in the present discussion
M_{OT}	axisymmetric pitch moment acting at the origin of the model coordinate system (O)
M_x, M_y, M_z	components of the external moments vector, $\mathbf{M}$, in the body coordinate system
m	vehicle mass
N	normal force
N_T	axisymmetric normal force acting at the origin of the model coordinate system (O)
$\mathbf{p}$	linear momentum vector
p, q, r	components of the vehicle rotation rate vector, Ω, in the body coordinate system

Symbols

q_∞	dynamic pressure
$\mathbf{r}_{CM}$	vector from the origin of the model coordinate system (O) to the origin of the body coordinate system (center of mass)
$\mathbf{r}_m$	position vector of an infinitesimal mass element dm
S	reference area
$[T^{BP}]$	rotation matrix from the body to the planet coordinate system and the vehicle-carried planet coordinate system
	element i, j = 1,2,3 of the matrix $[T^{BP}]$
t	time
u, v, w	components of the vehicle center of mass velocity vector, **V**, in the body coordinate system
u_∞, v_∞, w_∞	components of the vehicle center of mass airspeed vector, $\mathbf{V}_\infty$, in the body coordinate system; same as (u, v, w) when there is no wind
V	velocity vector of the vehicle center of mass; same as $\mathbf{V}_\infty$ when there is no wind
$\mathbf{V}_\infty$	airspeed vector of the vehicle center of mass; same as **V** when there is no wind
V_∞	airspeed
W	vehicle weight
X_P, Y_P, Z_P	planet coordinate system axes

Symbols

X_V, Y_V, Z_V	vehicle-carried planet coordinate system axes
x, y, z	body coordinate system axes
x_m, y_m, z_m	components of the position vector, $\mathbf{r}_m$, in the body coordinate system
Y	side force
α	angle of attack
α_T	total angle of attack
β	angle of sideslip
γ	flight path angle
ε_1, ε_2, ε_3, ε_4	Euler Parameter
ε_{1R}, ε_{2R}, ε_{3R}, ε_{4R}	renormalized Euler Parameter
θ	elevation (pitch) Euler Angle
ξ, η, ζ	model axes
ξ_{CM}, η_{CM}, ζ_{CM}	location of the vehicle's center of mass in the model axes
ρ	atmospheric density
ϕ	bank (roll) Euler Angle
$\phi_{\alpha T}$	total angle of attack clock angle
X	generic state variable
ψ	azimuth (yaw) Euler Angle
$\mathbf{\Omega}$	vehicle rotation rate vector

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